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# Duality, Strings and Supergravity: a Status Report

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## Abstract

We will report on recent advances in the understanding of non-perturbative interconnections between different string dualities. Weak-strong coupling duality (S-duality) and T-duality (symmetry under compactification on dual tori) allows one to compare and explore the strong coupling regime of seemingly unrelated theories. These theories naturally merge in a quantum version of supergravity called M-theory. The dynamical role of ‘branes’ of different nature and the new dynamical tool of (M)atrix formulation of M-theory will be briefly mentioned.

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# Duality, Strings and Supergravity: a Status Report

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## Abstract

We will report on recent advances in the understanding of non-perturbative interconnections between different string dualities. Weak-strong coupling duality (S-duality) and T-duality (symmetry under compactification on dual tori) allows one to compare and explore the strong coupling regime of seemingly unrelated theories. These theories naturally merge in a quantum version of supergravity called M-theory. The dynamical role of ‘branes’ of different nature and the new dynamical tool of (M)atrix formulation of M-theory will be briefly mentioned.

## 1 Introduction

In the past three years, since the last ICMP held in Paris in 1994, stunning developments in non-perturbative quantum theories of basic forces have been taking place. These dramatic results occurred in two different but related types of quantum theories: supersymmetric Yang–Mills theories in four dimensions, [1] and superstrings [2]. These theories have in common two basic symmetries: 1) space-time supergravity, which controls both perturbative and non-perturbative effects, 2) “duality”, which allows relating different (or the same) theories in different coupling constant regimes and fundamental states to solitonic excitations.

These theories also share the property that a formal perturbative expansion can be defined for them, and “renormalization” can be used in order to extract finite answers from any perturbative calculation.

They enjoy non-renormalization theorems [3] depending on the degree of supersymmetry of the vacuum around which we define the quantum perturbative series.

Let us recall the spin content of the light states: the gauge theory includes massless quanta with spin 0, 1/2, 1; superstrings include massless quanta with spin 0, 1/2, 1, 3/2 and 2; in a suitable limit ( $\alpha' \rightarrow 0$ , fixed gauge-coupling) superstrings reproduce ordinary gauge theories.

Exact non-perturbative results of SQCD have been obtained. Seiberg and Witten [4] suggested an exact expression for the low-energy effective action for the Coulomb phase of an  $N = 2$  SU(2) SYM theory, which may be regarded as an extension of the Georgi–Glashow SU(2) gauge theory.

Electric–magnetic duality plays a crucial role to solve the theory, to compute strong coupling phases of this theory (where massless monopoles and dyons appear), and to prove colour confinement through a magnetic Higgs mechanism with a monopole condensation (analogous to Meissner effects in superconductors).

The solution of the problem is possible thanks to non-renormalization theorems, making the complete perturbative computation affordable.

The non-perturbative part, conjecturally due to processes from multi-instanton transitions, is obtained from a mathematical hypothesis that identifies electric and magnetic massive states with windings  $(n, m)$  of a genus 1 (torus) elliptic Riemann surface (genus  $n$  for an SU( $n$ ) gauge-theory as shown by Klemm, Lerche, Theisen and Yankielowicz and by Argyres and Faraggi [5]). Two topologically different cycles correspond to electric and magnetic charges. Quantum massive BPS states  $\psi(n, m)$  correspond to distinct topological configurations  $(n, m)$  of the elliptic surface, in particular  $\psi(1, 0) = W$ -boson,  $\psi(0, 1) =$  monopole,  $\psi(-1, +1) =$  dyon.

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## 2 Electric–Magnetic Duality and Supersymmetry

Let us recall the duality of Maxwell equations

$$\begin{aligned}\nabla \cdot (E + iB) &= \rho_e + i\rho_m = \rho \\ \nabla \wedge (E + iB) - i\frac{\partial}{\partial t}(E + iB) &= J_e + iJ_m = J \\ L &= \text{Re}(E + iB) \cdot (E + iB) = E^2 - B^2 .\end{aligned}$$

Here  $E, B$  are the electric and magnetic fields;  $\rho_e (\rho_m), J_e (J_m)$  denote electric (magnetic) charge density and current, respectively. A topological term  $E \cdot B$  may eventually be added to  $L$ . The physical observables such as the energy density  $(E + iB) \cdot (E - iB) = E^2 + B^2$ , and the momentum density  $(E + iB) \wedge (E - iB)$  are invariant under (continuous)  $U(1)$  duality rotations:

$$(E + iB) \rightarrow e^{i\varphi}(E + iB) , \quad \rho \rightarrow e^{i\varphi}\rho , \quad J \rightarrow e^{i\varphi}J .$$

In particular the  $Z_2$  symmetry, which is the remnant of  $U(1)$ , acting on discrete charged states, exchanges electric with magnetic fields  $E \rightarrow B$  and  $B \rightarrow -E$ , and electric and magnetic charges  $q \rightarrow g, g \rightarrow -q$ , accordingly.

The simultaneous occurrence of electric and magnetic sources implies a charge quantization, which reads:

$$\begin{aligned}qg &= 2\pi k \\ (\text{Dirac 1931}) [6] \text{ (for monopoles)}\end{aligned}$$

and

$$\begin{aligned}q_1 g_2 - q_2 g_1 &= 2\pi k \\ (\text{Schwinger, Zwanziger 1968}) [7] \text{ (for dyons)}.\end{aligned}$$

In the Coulomb phase the Georgi–Glashow  $SU(2)$  gauge theory has a monopole with mass ('t Hooft, Polyakov 1974) [8]:

$$\begin{aligned}M_{\text{monopole}} &\geq 1/\lambda\langle\phi\rangle, \\ (\text{Bogomolny bound, 1975}) [9]\end{aligned}$$

while the classical vector-boson mass is  $M_W = \lambda\langle\phi\rangle$ . In the Prasad–Sommerfeld (1976) limit [10] (supersymmetry),  $M_{\text{monopole}} = 1/\lambda\langle\phi\rangle$  satisfies the duality conjecture (Montonen, Olive 1977) [11]:

$$M^2(q, g) = M^2(q^2 + g^2) = \langle\phi\rangle^2(q^2 + g^2) .$$

This generalizes when a topological term  $\theta E \cdot B$  is included by defining a complex parameter  $\tau = \theta + i/\lambda^2$  and then writing:

$$M^2(\phi, \tau, n, m) = \frac{|\phi|^2}{\text{Im}\tau} |n + \tau m|^2$$

invariant under  $SL(2, Z)$ :

$$\begin{aligned}Z_2 \quad : \quad \tau &\rightarrow \frac{-1}{\tau} , \quad n \rightarrow m , \quad m \rightarrow -n \\ \theta - \text{shift} \quad : \quad \tau &\rightarrow \tau + 1 , \quad n \rightarrow n - m , \quad m \rightarrow m .\end{aligned}$$

This means that the dual theory obtained by a  $Z_2$  symmetry  $E \rightarrow B, B \rightarrow -E$  has  $\tau_d = -1/\tau, n_D = m, m_D = -n$ . The  $N = 4$  supersymmetric Yang–Mills theory realizes the Montonen–Olive duality conjecture [11]. The theory has an exact  $SL(2, Z)$  symmetry [12], [4], which is possible in virtue of a vanishing  $\beta$  function, in the full quantum theory. Electric states are fundamental, while magnetic states are solitons in the theory  $T$ , but their role is reversed in the dual theory  $T_D$  [13].

Seiberg and Witten [4] extended the duality to  $N = 2$ , SYM quantum field theories undergoing renormalization ( $\beta \neq 0$ ), which gives corrections to a ‘holomorphic prepotential’,  $F(\phi)$ ; this is the appropriate tool to build up  $N = 2$  effective actions. The BPS states (which lie in hypermultiplets) have mass  $M(\phi, n, m, \lambda) \propto |\phi n + F_\phi m|$ , where [14]  $F(\phi) = (i/2\pi)\phi^2 \ln(\phi^2/\lambda^2) + \dots$  (the dots denote the non-perturbative contributions).

They also extended the duality conjecture. This came by identifying the pair  $(\phi, F_\phi)$  with the periods of an hyper-elliptic surface, which allows us to give a closed expansion for  $F(\phi)$ . As a result, at strong coupling  $\phi^2/\lambda^2 = \pm 1$ , one gets a massless monopole  $(0,1)$  and a dyon  $(-1, +1)$ .

The dual  $U(1)$  magnetic theory is weakly coupled in the strong coupling of the electric theory and describes a magneto-dynamic of a charged monopole. In the weakly coupled magnetic Higgs phase, monopole condensation describes confinement of the original (strongly coupled) electric theory. It is worth mentioning that, for BPS states, their mass appears in the central extension of the supersymmetry algebra (Haag–Lopuszanski–Sohnius) [15] and this allows one, using supersymmetry [16], to compute their mass in terms of the low-energy data. The duality has been further extended to  $N = 1$  super-Yang–Mills theories [17], in particular to SQCD with colour group  $SU(N_c)$  and  $N_f$  flavours. This theory has an anomaly-free global symmetry:

$$SU_L(N_f) \times SU_R(N_f) \times U(1)_B \times U(1)_R .$$

Seiberg suggested that there is a non-Abelian Coulomb phase for  $3/2N_c < N_f < 3N_c$ . At the non-trivial infra-red fixed point, the theory of quarks and gluons has a dual description in terms of an interacting conformal invariant theory with magnetic gauge group  $SU(N_f - N_c)$  and  $N_f$  flavours. Quarks and gluons are solitons in the dual picture.

### 3 Supergravity, Strings and M-Theory

Duality symmetries in the context of supergravity theories [18], further extended to superstrings [19], allow us to prove exact equivalences of different string theories [20]–[22], to obtain a dynamical understanding of the Seiberg–Witten conjecture in the point-particles limit [23],[24] and finally to possibly merge these theories in the context of M-theory, a supposedly existing quantum theory of membranes and five-branes [25], [26], whose low-energy effective action is 11- $D$  supergravity [27].

There are five known types of superstring theories in 10 dimensions [2]:

Type	Gauge group
Type I	$SO(32)$
Heterotic	$SO(32), E_8 \times E_8$
Type IIA	$U(1)$
Type IIB	None

The first three have  $N = 1$  supersymmetry, while the last two have  $N = 2$ , non-chiral type IIA and chiral IIB. There is also a conjectured M-theory in eleven dimensions [22] (no gauge group). Upon reduction on a circle, this is equivalent to type IIA, at the non-perturbative level. A further speculative theory may exist in twelve dimensions, which gives, upon reduction on a two-torus, the type IIB theory [28].

The previous theories, and their compactification to lower dimensions, reduce at low energies to supergravity theories in diverse dimensions [29] with underlying supersymmetry algebras as classified by Nahm [30]. In the highest and lowest dimensions of interest, we have for instance:

$$\begin{aligned}
D = 11, N &= 1, 128_{\text{boson}} + 128_{\text{fermions}} \\
&\quad (b = 44 + 84, f = 128) \\
D = 10, N &= 1 \text{ (chiral)} \\
N &= 1 \text{ (matter)} (G = E_8 \times E_8, SO(32)) \\
N &= 2: \text{ type A (non-chiral),} \\
&\quad \text{type B (chiral)} \\
D = 4, N &= 1 \text{ (chiral): obtained as } M\text{-theory} \\
&\quad \text{on } M_7 = CY_3 \times S_1/Z_2 \\
N &= 8 \text{ (non-chiral) } (b = 56 + 70 + 2, \\
&\quad f = 112 + 16): \text{ obtained as} \\
&\quad M\text{-theory on } T_7(U(1)^{28} \text{ gauge} \\
&\quad \text{group)} \\
&\quad \text{or on } S_7(SO(8) \text{ gauge group}) .
\end{aligned}$$

Let us summarize some of the main basic results of the years 94–97, in the context of string theory and its non-perturbative regime.

- 1) The Seiberg–Witten solution of rigid  $N = 2$  theory generalizes to heterotic-type II duality relating  $K_3 \times T_2$  vacua of heterotic to Calabi–Yau vacua of type II strings [31], [32], [33], [34]. The second quantized mirror symmetry [31] gives exact non-perturbative results in  $N = 2$  superstrings,  $D = 4$ . In particular, duality relates world-sheet instanton effects on the type II side to space-time instantons on the heterotic side [23], [35]. Dual pair heterotic-type II theory constructions were proposed [36].
- 2) The implication of string–string duality in six dimensions for S–T duality at  $D = 4$  was first shown by Duff [20], and  $U$ -duality as a non-perturbative symmetry of different string theories was formulated by Hull and Townsend [21].
- 3) Witten [22] proved the equivalence of different string theories in higher dimension and the duality of type IIA at strong coupling with 11- $D$  supergravity at large radius (M-theory on  $M_{10} \times S_1$ ). Type IIB is self-dual at  $D = 10$  (SL(2,  $Z$ ) duality) [37].
- 4) The  $E_8 \times E_8$  heterotic string at strong coupling is dual to the M-theory on  $M_{10} \times S_1/Z_2$  (Horava–Witten) [38].
- 5) The SO(32) Type I and SO(32) heterotic at  $D = 10$  are interchanged by weak–strong coupling duality (Polchinski–Witten) [39].
- 6) Open strings naturally arise, by the mechanism of tadpole cancellations, as sectors of type IIB closed strings on orientifolds [40], [41]. Their end-points end on  $D$ -branes [42], carrying R–R charges. Phase transitions in six dimensions are possible [43], and evidence for a non-perturbative origin of gauge symmetries [44] was substantiated [45].
- 7) T-duality at  $D = 9$  relates type IIA and type IIB theories, as well as SO(32) and  $E_8 \times E_8$  heterotic strings in their broken phase SO(16) $\times$ SO(16) [39].
- 8) M-theory and strings may undergo a further unification in twelve dimensions ( $F$ -theory) [28].
- 9) M-theory in the infinite momentum frame is equivalent to the large- $N$  limit of certain Yang–Mills theories [46], [47], [48].
- 10) Brane dynamics in M-theory reproduces non-perturbative aspects of rigid super-Yang–Mills theories and in particular the Seiberg–Witten result [49].

In the next sections we will describe some aspects of the above results, namely some non-perturbative properties of superstring theories. In particular in section 4 we will describe duality symmetries of M-theory in various dimensions and their connections to S- and T-dualities of string theories, in section 5 some particular aspects of the physics of extremal BPS black-holes, in section 6 the Matrix-Model formulation of M-theory and finally in section 7 an application of these non-perturbative results to coupling-constant unification.

## 4 Dualities in Lower Dimensions

A major role in exploring non-perturbative properties of supersymmetric theories when compactified to dimensions  $D < 10$  is played by the so-called U-duality group [50]. This group combines the S-type duality [51], *i.e.* the coupling-constant inversion, with the so-called T-duality, *i.e.* the geometrical symmetries of the manifold of compactification, augmented with some discrete translational symmetries, which appear in a natural way in the perturbative string formulation [37].

For toroidally compactified strings, where the number of unbroken supersymmetries does not decrease in the process of compactification, these discrete groups are always subgroups of the non compact continuous duality groups of the effective maximally-extended supergravity field theory [52].

These groups play an important role in the non-perturbative study of these theories.

Two important applications, discussed in the subsequent sections, are the black-hole entropy of BPS states preserving one supersymmetry and the Matrix-model formulation of M-theory.

For Type II string theory compactified on a  $d$ -dimensional torus  $T^d$ , the U-duality group is  $E_{d+1}(\mathbb{Z})$ , which is a discrete subgroup of the maximal non compact form of the  $E_{d+1}$  group. For  $d = 1, 2, 3, 4$ , these groups are  $GL(2, \mathbb{R})$ ,  $SL(3, \mathbb{R}) \times SL(2, \mathbb{R})$ ,  $SL(5, \mathbb{R})$ ,  $O(5, 5)$  respectively. For  $d = 5, 6$  we encounter the exceptional groups  $E_{6(6)}$  and  $E_{7(7)}$ .

Note that these groups have rank  $d + 1$  and, being maximally non compact, their Cartan subalgebra can be taken to be  $O(1, 1)^{d+1}$ . These generators correspond to the string coupling and to the  $d$  circles in  $T^d \sim (S_1)^d$ .

Under S–T-duality, the group  $E_{d+1} \supset O(1,1) \times O(d,d)$  at  $d < 6$  ( $E_7 \supset SU(1,1) \times O(6,6)$  at  $d = 6$ ) [53], [54].

The corresponding discrete subgroup is  $Z_2 \times O(d,d;\mathbb{Z})$  at  $d < 6$  ( $SL(2,\mathbb{Z}) \times O(6,6)$  at  $d = 6$ ), *i.e.* the coupling-constant inversion and the T-duality of ten-dimensional strings compactified on  $T^d$ .

This subgroup does not mix R-R and N-S states. The remaining transformations in  $E_{d+1}$  mix R-R and N-S states and generate U-dualities. These transformations mix D-branes [55] with N-S branes and correspondingly R-R charges with N-S charges. These symmetries are non-perturbative in nature.

To understand the charge content of a generic multicharge configuration, it suffices to decompose the charge vector of the U-duality group with respect to S–T-dualities.

Let us consider, as an example, the  $D = 5$  and  $D = 4$  black-holes, corresponding to 0-branes at  $D = 5$  and  $D = 4$ .

The generic charge vector is in the **27** of  $E_{6(6)}$  at  $D = 5$  and in the **56** of  $E_{7(7)}$  at  $D = 4$ .

Its decomposition under S-T duality is:

$$\begin{aligned} 27 &\rightarrow 16_1 + 10_{-2} + 1_4 \\ 56 &\rightarrow (1, 32) + (12, 2). \end{aligned} \tag{1}$$

The R-R charges are  $O(d,d)$  spinors, therefore there are 16 charges at  $D = 5$  and 32 (dyonic) charges at  $D = 4$ . These charges correspond to D-branes configurations wrapped and reduced in different ways.

## 5 Black-Hole Entropy and Bekenstein-Hawking Formula

The U-duality classification of black-holes, as well as of other extended objects, plays a central role in the determination of the entropy of a multicharge configuration.

The entropy can be computed either with a microscopic calculation, using D-brane techniques, by counting microstates [56], [57], or using the Bekenstein–Hawking area formula [58] from the low-energy supergravity effective field theory [59], [60], [61], [62], [63].

Since the entropy is topological in nature [64] and does not depend on the moduli fields, *i.e.* the v.e.v. of scalar fields at infinity, it can only depend on quantized charges. Because of U-duality it must be a U-invariant combination of the charges.

Indeed the black-hole entropy, at  $D = 5$  and  $D = 4$ , is proportional to the square root of the unique cubic and quartic invariants of the fundamental representations of  $E_{6(6)}$  and  $E_{7(7)}$  respectively [61], [65].

This result can be obtained, in the supergravity framework, by computing the horizon area of an extremal BPS black hole preserving 1/8 of supersymmetry. The actual value is proportional to  $M_{extr}^{3/2}$  at  $D = 5$  and to  $M_{extr}^2$  at  $D = 4$ , where  $M_{extr}$  is the value of the BPS mass extremized in the moduli space [61]. The extrema occur for moduli at rational values in terms of the quantized electric and magnetic charges [66]. The latter is the BPS mass of the so-called double extreme black holes [67].

To make contact with the D-brane microscopic calculation, it is useful to compute the cubic and quartic invariants in the so-called “normal frame” [68], *i.e.* in the frame where the charge vector is represented by a skew diagonal matrix.

Here we will briefly describe the 5- $D$  case [69].

At  $D = 5$ , in terms of the three eigenvalues  $e_i$  of a traceless  $8 \times 8$  matrix, the  $27^3$  invariant reduces to:

$$I(27^3) = s_1 s_2 s_3 \tag{2}$$

where

$$e_1 = s_1 + s_2 - s_3, \quad e_2 = s_1 - s_2 + s_3, \quad e_3 = -s_1 + s_2 + s_3 \tag{3}$$

Taking the case of Type II on  $T^5$  we can choose  $s_1$  to correspond to a solitonic five-brane charge,  $s_2$  to a fundamental string winding charge along some direction and  $s_3$  to Kaluza-Klein momentum along the same direction.

We can see that in this specific example one breaks 1/8 supersymmetry if  $s_1 s_2 s_3 \neq 0$ , 1/4 if  $s_1 s_2 \neq 0$ ,  $s_3 = 0$ , 1/2 if  $s_1 \neq 0$ ,  $s_2 = s_3 = 0$ .

The case of enhanced supersymmetry corresponds to the invariant combinations  $I_3 = 0$  and  $\frac{\partial I_3}{\partial e_i} = 0$  and selects specific orbits of the **27**, *i.e.* the generic orbit ( $I_3 \neq 0$ ) preserving 1/8 supersymmetry is  $E_{6(6)}/F_{4(4)}$ , while the critical orbit, preserving 1/2 supersymmetry, is  $E_{6(6)}/O(5, 5) \otimes T_6$  [70], [71].

The basis chosen in the above example is S-dual to the D-brane basis usually chosen for describing black holes in Type IIB on  $T_5$ . All other bases are related by U-duality to this particular choice. We also observe that the above analysis relates the cubic invariant to the picture of intersecting branes since a three-charge 1/8 BPS configuration with non vanishing entropy can be thought as obtained by intersecting three single charge 1/2 BPS configurations [72], [73], [74]

By using the S–T-duality decomposition we see that the cubic invariant reduces to  $I_3(27) = 10_{-2}10_{-2}1_4 + 16_116_110_{-2}$ . The 16 correspond to D-brane charges, the 10 correspond to the 5 KK directions and winding of wrapped fundamental strings, the 1 correspond to the N-S five-brane charge.

We see that to have a non-vanishing area we need a configuration with three non-vanishing N-S charges or two D-brane charges and one N-S charge.

Unlike the 4- $D$  case, it is impossible to have a non-vanishing entropy for a configuration only carrying D-brane charges.

## 6 Matrix Model Formulation of M-Theory

In the last year a proposal for a non-perturbative formulation of M-theory has been made in which M-theory compactified on  $T^d$  is related to a  $d + 1$  supersymmetric Yang–Mills theory compactified on a dual torus  $\tilde{T}^d$  [46].

Since the Yang–Mills coupling  $g^2$  in  $d + 1$  dimensions has dimensions  $m^{3-d}$ , we may introduce a dimensionless running coupling constant as  $\hat{g}^2(\mu) = g^2\mu^{d-3}$ . In particular the coupling strength at scales of the order of the torus size  $\tilde{L}$  is  $\hat{g}^2(\mu) = g^2\tilde{L}^{3-d}$ .

Then we can study the ultraviolet (infra-red) properties of the theory by decreasing (increasing)  $\tilde{L}$ . For  $d < 3$ , going to strong dimensionless coupling probes the infra-red behaviour of super-Yang–Mills theory. For  $d > 3$  it probes the ultraviolet.

Matrix theory is defined as the large  $N$  limit of a  $U(N)$  super-Yang–Mills theory with the infinite momentum limit in which the 11-th direction is chosen to be the longitudinal direction. The theory is assumed to be compactified in this direction on a circle of size  $R$ . When  $d$  transverse directions are compactified on a  $d$ -torus then they will be related to M-theory compactified on a dual torus. In the original discussion the starting point was the theory of D0-branes in IIA theory when the compact direction is identified with the longitudinal direction [46], [47].

The starting point is that M-theory on  $S_1$  of size  $L$  is equivalent, through the Matrix-Model construction, to uncompactified perturbative Type IIA string theory in the limit of shrinking torus size.

The limit  $L \rightarrow 0$  corresponds to strongly coupled Yang–Mills theory with  $\hat{g}^2 = (2\pi)^4(\ell_{11}/L)^3$  and string coupling constant  $g_s^2 = 2\pi/\hat{g}^2$  ( $\ell_{11}$  is the 11- $D$  Planck length).

The relation between the dual radius  $\tilde{L}$  on which Yang–Mills theory is compactified and the M-theory radius  $L$  is:  $\frac{\tilde{L}}{4\pi^2} = \frac{\ell_{11}^3}{RL}$  ( $R$  is the radius of the compactified longitudinal direction).

In particular the limit of strongly coupled Yang–Mills theory becomes the weakly coupled IIA string theory in  $D = 10$ .

In the case of the two torus, the U-duality group is  $SL(2, \mathbb{Z}) \times Z_2$ , and  $SL(2, \mathbb{Z})$  corresponds to the geometrical symmetry of the two-torus.

In the case of the three-torus we have 4- $D$  Yang–Mills theory. The U-duality group is  $SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z})$ , where the first factor can be interpreted as the geometrical symmetry and the second factor to the electric-magnetic duality of four dimensional super-Yang–Mills theory.

The analysis becomes more subtle when  $d > 3$ . First, the Yang–Mills theory becomes non renormalizable. Moreover the electric and magnetic fluxes of super-Yang–Mills theory do not complete the U-duality multiplets; however proposals have recently been made for the cases  $d = 4$  and  $d = 5$  in terms of fixed-point six-dimensional (2, 0) superconformal field theories and of N-S five-branes [48].

## 7 Gauge Couplings Unification

New predictions of non-perturbative string theories can be derived from these non perturbative relations between the five seemingly different superstring theories. As a circumstantial example [75], strongly coupled heterotic string meets the agreement of  $\alpha_{GUT}$  as measured (at LEP) from low-energy data.

In weakly coupled heterotic string, compactified on a Calabi–Yau threefold of size  $V \approx M_{GUT}^{-6}$  with  $G_N = \frac{e^{2\phi}(\alpha')}{64\pi V}$ ,

$$\alpha_{GUT} = e^{2\phi}(\alpha')^3/16\pi V \rightarrow G_N = \alpha_{GUT}\alpha'/4.$$

If  $e^{2\phi} \leq 1$ ,  $G_N \geq \alpha_{GUT}^{4/3}/M_{GUT}^2$ , which is too large with respect to experiment.

In type I string (weak coupling),

$$\begin{aligned} \alpha_{GUT} &= \frac{e^{\phi_I}(\alpha')^3}{16\pi V}, \quad G_N = \frac{e^{2\phi_I}(\alpha')}{64\pi V} \\ &\rightarrow G_N = e^{\phi_I}\alpha_{GUT}\alpha'/4. \end{aligned}$$

Here,  $G_N$  can be small.

In the M-theory set-up ( $\kappa$  is the 11- $D$  gravitational coupling and  $\rho$  the compactification radius):

$$G_N = \frac{k^2}{16\pi^2 V \rho}, \quad \alpha_{GUT} = \frac{(4\pi k^2)^{2/3}}{2V} \ll 1.$$

So no disagreement with the experimental input exists in principle.

Finally, supersymmetry breaking can be described in a natural way both through a strongly coupled hidden gauge sector leading to gaugino condensation [76], [77] and through the no-scale structure [78] of M-theory. The decompactification problem may be avoided [79]–[81].

## Conclusions

In this status report we have summarized the tremendous advances that have been made over the last three years in our understanding of non-perturbative phenomena of quantum theories encompassing the fundamental interactions. These results heavily used mathematical concepts such as algebraic geometry [82], space-time symmetries such as supersymmetry, and topological configurations such as BPS solitonic states of different nature [83]. The old idea of electric–magnetic duality has revealed new insights in the dynamics of these theories with major advances achieved between the Paris and Brisbane ICMP conferences.

We hope that further progress and new directions, such as the understanding of the origin of symmetry breaking and the real spectrum of elementary particles, can be conceived in the years to come.

It is plausible that even more stunning developments will be reported at the Fourteenth ICMP Conference, to be held in Postdam at the end of this millenium.

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